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Theoretical Explanations of the *g* Factors for Ni³⁺ Ions in KMgF₃ and CsCaF₃ Crystals

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ABSTRACT By using the optical spectral parameters of (NiF₆)³⁻ clusters in KMgF₃:Ni³⁺ and CsCaF₃:Ni³⁺ crystals estimated from the observed values in pure K₃NiF₆ crystal, the electron paramagnetic resonance (EPR) *g* factors for KMgF₃:Ni³⁺ and CsCaF₃:Ni³⁺ are calculated from the second-order perturbation formula (with the high-spin ground orbital state) based on the cluster approach. The results are in reasonable agreement with the observed values. The spin transition from the low-spin state in pure K₃NiF₆ crystal to the high-spin state in KMgF₃:Ni³⁺ and CsCaF₃:Ni³⁺ crystals owing to the slight increase of Ni³⁺ – F⁻ distance is discussed.

KEYWORDS crystal- and ligand-field theory, CsCaF₃, electron paramagnetic resonance, KMgF₃, Ni³⁺, spin transition

INTRODUCTION

In crystal theory, according to the Tanabe-Sugano diagram,^[1] the ground orbital state (which is related to optical spectra) for some 3dⁿ ions in crystals depends upon the ratio $|Dq/B|$ (where Dq is the cubic field parameter and B is the Racah parameter).^[1-3] For example, for 3d⁷ ions in octahedral clusters, when $|Dq/B| > 2.2$, the ground orbital state is the low-spin ($S = 1/2$) state 2E .^[1,2] In this strong field case, the average EPR \bar{g} factor is close to 2.^[3,4] Whereas if $|Dq/B| < 2.2$, the ground orbital state should be the high-spin ($S = 3/2$) state $^4T_1(F)$;^[1,2] the \bar{g} factor in this weak field case is close to 13/3 (≈ 4.33).^[3,4] For an (NiF₆)³⁻ cluster in pure K₃NiF₆ crystal, the optical spectra studies suggested that $Dq \approx -1620 \text{ cm}^{-1}$, $B \approx 703 \text{ cm}^{-1}$, and so $|Dq/B| \approx 2.3$.^[5] Thus, its ground orbital state is the low-spin 2E .^[5] This point was supported by the EPR experiments for (NiF₆)³⁻ clusters in the pure crystals K₃NiF₆, Na₃NiF₆, Cs₂NaNiF₆, Cs₂KNiF₆, and Rb₂KNiF₆.^[6] Considering that the cubic field parameter Dq decreases by the relation $Dq \propto R^{-n}$ (where $n \approx 3.5 \sim 6.5$ ^[7,8]), and the Racah parameter B increases slightly with the increasing metal-ligand (e.g., Ni³⁺ – F⁻ here) distance R because of the decreasing covalence reduction effect,^[9,10] the value of $|Dq/B|$ for the (NiF₆)³⁻ clusters should decrease with the increase of distance R and may be smaller than 2.2 in some systems. Thus, the ground orbital state changes from the low-spin to high-spin state. For Ni³⁺ in cubic perovskites KMgF₃ and CsCaF₃, since the ionic radii r_b of the replaced host ions Mg²⁺ ($r_b \approx 0.86 \text{ \AA}$ ^[11]) and Ca²⁺ ($r_b \approx 1.14 \text{ \AA}$ ^[11]) are larger than the radius

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r ($\approx 0.74 \text{ \AA}$ in the high-spin state^[11]) of the impurity Ni^{3+} , the $\text{Ni}^{3+} - \text{F}^-$ distance R in the two crystals should be larger than that in the pure crystal. Thus, the values of $|Dq/B|$ may be smaller than 2.2, and the ground state may be of high spin. The observed EPR g factors for Ni^{3+} in both crystals^[12,13] support this point. No theoretical calculations for these g factors of Ni^{3+} in KMgF_3 and CsCaF_3 crystals were made. In this article, these g factors are calculated from the second-order perturbation formulas based on the cluster approach. In the calculations, the optical spectra parameters (Dq and B) are estimated reasonably from those observed in pure K_3NiF_6 crystals and the above rules related to optical spectral parameters Dq , B , and metal-ligand distance R . The results are discussed.

CALCULATION

For $3d^7$ ions in a weak octahedral crystal field, the ground orbital state ${}^4\text{T}_1(\text{F})$ splits into three Kramers doublets ($J' = 5/2, 3/2$, and $1/2$). The lowest Kramers doublet is $|\pm 1/2\rangle$.^[4] Thus, considering the covalency effect in $3d^n$ clusters, the second-order perturbation formula of a g factor based on the cluster approach for $3d^7$ ions in cubic crystals can be expressed as^[14]

$$g = \frac{10}{3} + \frac{2}{3}k\alpha + \frac{5k'\zeta'\varepsilon^2}{2\Delta_F}, \quad (1)$$

where k and k' are the orbital reduction factors. ζ' (and ζ) are the spin-orbit (SO) coupling parameters. Δ_F is the energy separation between the excited state ${}^4\text{T}_2(\text{F})$ and the ground state ${}^4\text{T}_1(\text{F})$, which can be calculated from the optical spectra parameters. α is the effective Lande factor and reads

$$\alpha = \frac{3}{2}\varepsilon^2 - \tau^2, \quad (2)$$

in which the parameters ε and τ are related to the interaction (or admixture) between ${}^4\text{T}_1(\text{P})$ and ${}^4\text{T}_1(\text{F})$ and can be obtained from the optical spectra parameters by using the relationships^[14]

$$\varepsilon^2 + \tau^2 = 1, \quad \frac{\tau}{\varepsilon} = \frac{-4Dq}{15B - 6Dq}. \quad (3)$$

According to the cluster approach, the parameters ζ , ζ' , k , and k' can be written as^[14,15]

$$\begin{aligned} \zeta &= N_t(\zeta_d^0 + \lambda_t^2 \zeta_p^0/2), \\ \zeta' &= \sqrt{N_t N_e}(\zeta_d^0 + \lambda_t^2 \zeta_p^0/2), \\ k &= N_t[1 - 2\lambda_t S_{dp}(t_{2g}) + \lambda_t^2/2], \\ k' &= \sqrt{N_t N_e}[1 - \lambda_t S_{dp}(t_{2g}) - \lambda_e S_{dp}(e_g) - \lambda_t \lambda_e/2], \end{aligned} \quad (4)$$

where ζ_d^0 and ζ_p^0 are, respectively, the SO coupling parameter of free $3d^n$ ion and that of free ligand ion. For the $(\text{NiF}_6)^{3-}$ clusters under study, $\zeta_d^0(\text{Ni}^{3+}) \approx 749 \text{ cm}^{-1}$,^[16] $\zeta_p^0(\text{F}^-) \approx 220 \text{ cm}^{-1}$,^[17] $S_{dp}(\gamma)$ ($\gamma = t_{2g}$ or e_g) is the group overlap integral. N_γ and λ_γ are the normalization coefficient and orbital mixing coefficient in the linear combination of atomic orbitals (LCAO) molecular orbital (MO). They can be obtained from the normalization relation^[14,18]

$$N_\gamma = [1 - 2\lambda_\gamma S_{dp}(\gamma) + \lambda_\gamma^2]^{-1} \quad (5)$$

and the approximate condition^[14,18]

$$f_\gamma \approx B/B_0 \text{ (or } C/C_0) \approx N_\gamma^2[1 + \lambda_\gamma^2 S_{dp}^2(\gamma) - 2\lambda_\gamma S_{dp}(\gamma)], \quad (6)$$

where B_0 and C_0 are the Racah parameters of a free $3d^n$ ion. For the free Ni^{3+} ion, we have $B_0 \approx 1195 \text{ cm}^{-1}$ and $C_0 \approx 4808 \text{ cm}^{-1}$.^[16]

The integral $S_{dp}(\gamma)$ of a $3d^n$ cluster can be calculated from the Slater-type self-consistent field (SCF) functions^[19,20] and the average metal-ligand distance R . Generally speaking, for $3d^n$ impurities in crystals, the impurity-ligand distance R is different from the corresponding distance R_H in the host crystal because of the size and/or charge mismatch. The impurity-ligand distance R in crystals can be obtained by analyzing the experimental (e.g., extended x-ray absorption fine structure (EXAFS) and superhyperfine constant experiments^[21]) data or by using the theoretical calculations (e.g., using the embedded-quantum-cluster approach^[22] or a lattice relaxation model^[23,24]). For the studied $\text{KMgF}_3:\text{Ni}^{3+}$ and $\text{CsCaF}_3:\text{Ni}^{3+}$ crystals, no such experimental and theoretical studies were done. According to the impurity-ligand distances R for Mn^{2+} in fluoroperovskites derived from the experimental superhyperfine constant A_s , EXAFS measurements and crystal-field spectrum analysis,^[21] an approximate formula

$$R \approx R_H + (r_i - r_b)/2 \quad (7)$$

TABLE 1 Mixing Parameters ε and τ , MO Coefficients, Orbital Reduction Factors, and SO Coupling Parameters for Ni^{3+} in KMgF_3 and CsCaF_3 Crystals

	ε	τ	N_t	N_e	λ_t	λ_e	k	k'	ζ'
$\text{KMgF}_3:\text{Ni}^{3+}$	0.9590	0.2835	0.7728	0.7814	0.5495	0.5564	0.8833	0.6436	604.5
$\text{CsCaF}_3:\text{Ni}^{3+}$	0.9634	0.2680	0.7758	0.7825	0.5430	0.5486	0.8856	0.6518	558.1

is suggested in the estimation of the impurity-ligand distance R in crystals.^[25] So, it is applied here. From the r_b , r_b given in the Introduction and $R_H \approx 1.994$ and 2.262 \AA ^[21] for KMgF_3 and CsCaF_3 , respectively, $R \approx 1.934$ and 2.062 \AA are obtained for $(\text{NiF}_6)^{3-}$ clusters in $\text{KMgF}_3:\text{Ni}^{3+}$ and $\text{CsCaF}_3:\text{Ni}^{3+}$ crystals. Thus, the integrals $S_{dp}(\gamma)$ can be calculated. They are $S_{dp}(t_{2g}) \approx 0.00728$, $S_{dp}(e_g) \approx 0.02682$ for $\text{KMgF}_3:\text{Ni}^{3+}$ and $S_{dp}(t_{2g}) \approx 0.00547$, $S_{dp}(e_g) \approx 0.02102$ for $\text{CsCaF}_3:\text{Ni}^{3+}$.

No optical spectra of $(\text{NiF}_6)^{3-}$ clusters in $\text{KMgF}_3:\text{Ni}^{3+}$ and $\text{CsCaF}_3:\text{Ni}^{3+}$ crystals were reported. We estimate reasonably the optical spectra parameters B , C , and Dq for both crystals from the optical spectra parameters of K_3NiF_6 crystal^[5] and the rules mentioned in the Introduction. Thus, from the $\text{Ni}^{3+} - \text{F}^-$ distances R in K_3NiF_6 , $\text{KMgF}_3:\text{Ni}^{3+}$, and $\text{CsCaF}_3:\text{Ni}^{3+}$ (note: for the pure K_3NiF_6 , considering that the metal-ligand distance in pure crystal is very close to the sum of ionic radii, it is taken that $R \approx 1.89 \text{ \AA}$, where $r(\text{Ni}^{3+}) \approx 0.70 \text{ \AA}$ in the low-spin state and $r(\text{F}^-) \approx 1.19 \text{ \AA}$ ^[11]), we have $Dq \approx -1410 \text{ cm}^{-1}$, $B \approx 708 \text{ cm}^{-1}$, and hence, $f_\gamma \approx 0.5925$ for $\text{KMgF}_3:\text{Ni}^{3+}$ and $Dq \approx -1280 \text{ cm}^{-1}$, $B \approx 715 \text{ cm}^{-1}$, and hence, $f_\gamma \approx 0.5980$ for $\text{CsCaF}_3:\text{Ni}^{3+}$.

By using these optical spectra parameters for Ni^{3+} in both crystals, the parameters ε and τ are calculated from Eq. (3), and the MO coefficients N_γ and λ_γ are obtained from Eqs. (5) and (6). Thus, the parameters k , k' , and ζ' can be calculated from Eq. (4). These calculated results are shown in Table 1.

Applying the parameters in Table 1 to Eq. (1), the g factors for Ni^{3+} ions in KMgF_3 and CsCaF_3 are calculated. They are compared with the observed values in Table 2.

TABLE 2 EPR g Factors for Ni^{3+} in KMgF_3 and CsCaF_3 Crystals

	$\text{KMgF}_3:\text{Ni}^{3+}$	$\text{CsCaF}_3:\text{Ni}^{3+}$
Calculation	4.163	4.186
Experiment	4.163 (1) ^a	4.183 ^b

^aReference 12.

^bReference 13.

DISCUSSION

In the above calculations, one can find that the values $|Dq/B| \approx 1.99$ and 1.79 for Ni^{3+} ions in KMgF_3 and CsCaF_3 crystals, respectively. Both values are smaller than 2.2, and so the ground orbital state of $(\text{NiF}_6)^{3-}$ clusters in the two crystals is, as suggested in the Introduction, the high-spin state rather than the low-spin state in K_3NiF_6 crystal. So, although there may be small errors in the estimated impurity-ligand distances in $\text{KMgF}_3:\text{Ni}^{3+}$ and $\text{CsCaF}_3:\text{Ni}^{3+}$ crystals because of the application of the approximate formula (i.e., Eq. (7)), the optical spectra parameters in the two crystals estimated from those in pure K_3NiF_6 crystal, and these distances are suitable. By using these parameters, the g factors of $\text{KMgF}_3:\text{Ni}^{3+}$ and $\text{CsCaF}_3:\text{Ni}^{3+}$ crystals are reasonably explained from the second-order perturbation formula based on the cluster approach for the high-spin $3d^7$ ions in cubic symmetry (see Table 2).

It should be pointed out that in the crystal-field theory, there are two theoretical methods—the complete diagonalization (of energy matrix) method (CDM)^[26–29] and the perturbation theory method (PTM),^[26–29]—in the calculations of spin-Hamiltonian parameters (including g factors) for $3d^n$ ions in crystals. Generally speaking, the PTM is simpler, and the CDM can provide more exact calculated results of spin-Hamiltonian parameters because it takes into account the contributions from all $3d^n$ excited states.^[26,27] However, for the g factors of $3d^n$ ions in crystals, the calculated results from both the CDM and PTM are close to each other.^[26–29] Recently, a fuller CDM^[27,30] was developed, in which in addition to the SO interaction in the conventional CDM, the spin-spin (SS) and spin-other-orbit (SOO) interactions are considered in the CDM calculations of spin-Hamiltonian parameters. Even so, the calculations for $3d^8$ and $3d^2$ ions in some crystals show that the contributions to g factors from the SS, SOO, and combined SO-SS-SOO interactions are

small.^[27,30] So, the calculated *g* factors of $\text{KMgF}_3:\text{Ni}^{3+}$ and $\text{CsCaF}_3:\text{Ni}^{3+}$ crystals using the simple PTM can be regarded as reasonable.

CONCLUSION

The theoretical calculations from the second-order perturbation formula (with the high-spin ground orbital state) based on the cluster approach confirmed that the ground state of the $(\text{NiF}_6)^{3-}$ clusters in $\text{KMgF}_3:\text{Ni}^{3+}$ and $\text{CsCaF}_3:\text{Ni}^{3+}$ crystals is the high-spin $^4\text{T}_1(\text{F})$ rather than the low-spin ^2E in K_3NiF_6 crystal. This spin transition from the low-spin to the high-spin for $(\text{NiF}_6)^{3-}$ cluster is due to the slight increase of $\text{Ni}^{3+} - \text{F}^-$ distance R .

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